

Labor Economics

August 2024

Please answer all three of the questions on this exam. You should plan to spend about one hour per question. Please write your answers for each question in a separate book.

1 ECON 250A: Search

Consider a worker searching sequentially for a job against an exogenous offer distribution F . The worker has CRRA utility function

$$U(w) = \frac{w^{1-\alpha}}{1-\alpha}$$

where $\alpha \in [0, 1)$ is the coefficient of relative risk aversion. If the worker doesn't find a job, they enjoy unemployment benefit $b > 0$. Next period utility is discounted at rate $\beta \in (0, 1)$. If the worker accepts a job offer, they are paid the same wage forever.

- Write the Bellman equation describing the value of a wage offer w
- Derive an expression characterizing the reservation wage w^*
- Use this expression to derive an expression for dw^*/db in terms of the objects $(\alpha, w^*, b, \beta, F)$. Use this expression to sign dw^*/db . Give intuition for your answer.
- Derive an expression for how the reservation wage changes with α . Use it to sign $\frac{dw^*}{d\alpha}$. Give intuition for your answer.
- Use your answers above to evaluate the sign of the cross-partial $\frac{\partial^2 w^*}{\partial b \partial \alpha}$. Give intuition for your answer.

2 ECON 250B: Contracts

Consider a setting where there are two risk-neutral workers and one risk-neutral manager. The workers choose their own levels of effort a_i and produce output x_i that is a noisy function of their effort:

$$\begin{aligned}x_1 &= a_1 + \epsilon_1 \\x_2 &= a_2 + \epsilon_2\end{aligned}$$

Where the errors ϵ_1 and ϵ_2 are independent, and identically distributed with mean 0 and variance σ^2 . The manager is only able to observe workers' output (i.e. not their effort).

The workers have a convex cost of effort function $c(a)$ and have outside option \bar{H} .

- [1 point] What is the efficient (first-best) level of effort? What is the expected total level of output produced when all workers exert the first-best level of effort?
- [1 point] **Piece-rates:** Suppose the manager decides to pay each worker using the same piece-rate function. What wage (if any) implements the first-best?
- Ranking Workers** Suppose instead the manager decides to pay workers based on the ranking of their output. The worker who produces more receives \bar{w} and the worker who produces less receives \underline{w} .
 - [1 point] What are some advantages and disadvantages of this scheme?
 - [1 point] Suppose worker 2 exerts effort a_2 . What is worker 1's expected payoff?
 - [1 point] Use $F(\cdot)$ to denote the CDF of $(\epsilon_2 - \epsilon_1)$, with corresponding density $f(\cdot)$. Write the first-order condition characterizing the level of effort that maximizes the worker's utility. Interpret this condition.

- (d) [1 point] Note that, in a symmetric equilibrium, both workers exert the same levels of effort $a_1^* = a_2^*$. In order for both workers to exert the first best level of effort, what must \bar{w} and \underline{w} be? Interpret these conditions. Hint: remember workers' participation constraints!
- (e) [1 point] Consider a setting where workers have a different cost of effort function $c_2(a)$ where $c_2'(a) < c_1'(a) \forall a$. How does the "spread" ($\bar{w} - \underline{w}$) in this setting compare to that in the base case?
- (f) [1 point] Suppose a new firm enters the market, raising workers' outside options to $\bar{H}^{new} > \bar{H}$. How does this affect the optimal choice of \bar{w} and \underline{w} ? Assume that the manager still wants to employ both workers.
4. **Empirical Literature:** [2 points] Relate your answers to the above to at least one of the empirical papers discussed in class.

3 ECON 250B: Unions

Consider a union with a utility function defined over the wage (W) and employment level (L). This utility function is

$$U(W, L) = (W - W_a)^\alpha L^{(1-\alpha)}$$

where W_a is the alternative wage of the workers and α is a parameter defined on the unit interval and represents the weight the union places on the wage relative to employment. Suppose the union is in a collective bargaining relationship with a firm with profits defined over the wage and employment. Profits (Π) are defined as the difference between revenues ($R = aL - L^2$) and labor costs ($C = WL$) so that

$$\Pi = R - C = (aL - L^2) - WL.$$

Assume that parameter $a > W$.

Consider three scenarios for wage and employment setting:

- The "right-to-manage" model. In this case the union sets the wage unilaterally and the firm chooses the employment level.
- A scenario where the union sets the wage and employment level to maximize its utility subject to the firm staying in business ($\Pi = 0$).
- A scenario where the firm sets employment at the level it would if the wage were equal to the alternative wage $W = W_a$ and the union chooses the wage to maximize its utility, again subject to the firm staying in business ($\Pi = 0$).

Here are your tasks.

- Solve for the equilibrium wage and employment levels in each scenario. [Hint: The algebra is a lot less complicated if you work with the log of the utility function.]
- Compare the wage and employment levels across the scenarios. Which scenario has the highest employment level? Which has the lowest? Which has the highest wage level? Which has the lowest?
- Does the ranking of the employment and wage levels depend on the parameter α ? If so, how? To the extent that the rankings depend on α , (briefly) provide intuition for the relationship.
- Discuss the efficiency properties of each scenario. Are any weakly efficient? Are any strongly efficient? Explain briefly.