Theory Field Examination

August 2024

Instructions: The field exam consists of two questions corresponding to Econ 206 and Econ 207A. Please answer both questions. You have a total of two hours. The exam is closed-book. There are a total of 100 points, and point totals for each sub-question are indicated. Good luck!

Question for 206 (50 points)

Consider a monopolist who has a unit mass of a homogeneous good to sell to a unit mass of infinitesimal buyers. Each buyer has a type $t \in [0,1]$, and the measure of types in the population has CDF F and density f. The payoff to a buyer with type t who consumes q units and makes a payment p is $tq - p$. A (direct) mechanism for the monopolist consists of a pair of allocation and payment rules, (q, p), where $q : [0,1] \to [0,1]$ describes quantity received, and $q : [0,1] \to \mathbb{R}$ describes payments to the monopolist.

So far, the setting coincides exactly with Myerson (1981). We now depart from the standard setting. Assume that prior to the arrival of the monopolist, the market was served by a government agency which provided each buyer with a quantity $a \in (0,1)$ of the good for free. Moreover, as a condition of allowing the monopolist to operate, the government requires that it make no buyer worse off than they were before its arrival. That is, the government imposes that the monopolist's mechanism (q, p) must satisfy the *status-quo* constraint:

$$q(t) - p(t) \geq at \quad \forall\, t \in [0,1].$$

1. (*10 points*) Formally state the monopolist's profit-maximization problem, including the usual Bayesian IC constraint, as well as the status-quo constraint introduced above. Write the monopolist's objective using the virtual values formulation of Myerson (1981).

2. (*15 points*) Prove that there exists an interval $[\underline{t}, \bar{t}] \subset [0,1]$ such that the status-quo constraint binds for $t \in [\underline{t}, \bar{t}]$ and is slack otherwise.

3. (*25 points*) Characterize the monopolist's profit-maximizing mechanism. What are the important qualitative features of the mechanism? (This question is intentionally open ended. Imagine that you are writing a paper on this topic, and think about what features of the mechanism you would highlight to tell an economic story).

References

R. B. Myerson. Optimal auction design. *Mathematics of operations research*, 6(1): 58–73, 1981.

Question for 207A (50 points)

1. (*25 points*) State and prove the rural hospitals' Theorem within the context of the matching with contracts model.

2. (*25 points*) Prove that in the school choice model, there is no strategy-proof mechanism that Pareto dominates the student optimal stable mechanism. Formally, let f denote the student optimal stable mechanism with respect to a fixed strict priority profile, then there is no mechanism $g \neq f$ such that for every preference profile R and every student $i \in N$, $g_i(R) R_i f_i(R)$.[1]

[1] For simplicity, you can assume that every school has one seat.