August 2022 Theory Field Exam – Economics 207a

Answer all of the questions below. Be as complete, correct, and concise as possible. You can use any results from class or elsewhere, with appropriate references, unless the question is asking you to prove that exact result.

1. Let X be a lattice, T be a partially ordered set, and $f : X \times T \to \mathbf{R}$. Suppose f is quasisupermodular in x and satisfies the strict single crossing property in $(x; t)$.[1] Show that every selection

$$x(t, S) \in \arg\max_{x \in S} f(x, t)$$

is nondecreasing in (t, S), that is, if $t' > t$ and $S' \geq_s S$, then $x(t', S') \geq x(t, S)$.

2. Let $\Gamma = (\mathcal{N}, (f_n, S_n, \geq_n)_{n\in\mathcal{N}}))$ be a game with strategic complementarities.[2]

Suppose that for each n, $S_n = [a_n, b_n] \subseteq \mathbf{R}$, where $a_n, b_n \in \mathbf{R}$ with $a_n \leq b_n$, and $\geq_n$ is the standard order in $\mathbf{R}$ for each n.

Consider the following learning dynamics, in which each player chooses a best response to an average of opponents' actions in previous periods. Fix an initial profile $x^0 \in S = S_1 \times \cdots \times S_N$, and for each $t \geq 1$ and each $n = 1, \ldots, N$, set

$$x_n^t = x_{*n}(y_{-n}^{t-1})$$

where $x_{*n}(x_{-n})$ denotes the smallest best response of player n to x_{-n}, and

$$y^0 = x^0, \quad x^t = (x_1^t, \ldots, x_N^t), \quad y^t = \frac{1}{2}(x^{t-1} + x^t) \ \forall t \geq 1$$

   a. Let $x^0 = (a_1, \ldots, a_N) = \inf S$. Show that $\{x^t\}$ converges.

   (Here convergence means with respect to the standard metric in $\mathbf{R}^N$. Hint: Recall that a bounded monotone sequence in $\mathbf{R}$ converges.)

   b. As in (a), let $x^0 = (a_1, \ldots, a_N) = \inf S$. Let $\hat{x} = \lim_t x^t$. Show that $\hat{x}$ is a Nash equilibrium.

   (As in (a), the limit is with respect to the standard metric in $\mathbf{R}^N$.)

[1] Recall f satisfies the strict single crossing property in $(x; t)$ if for all $x' > x$, $f(x', t) \geq f(x, t) \Rightarrow f(x', t') > f(x, t')$ for all $t' > t$.

[2] Recall that this means that $\mathcal{N} = \{1, \ldots, N\}$ for some finite integer N, and for each player $n \in \mathcal{N}$, (i) the action set S_n is a nonempty complete lattice with partial order $\geq_n$, and (ii) the payoff function f_n is order upper semicontinuous in (x_n, x_{-n}) and order continuous in x_{-n}, quasisupermodular in x_n, and satisfies the single crossing property in $(x_n; x_{-n})$.

Consider the following perfect-information extensive-form game: There are two players, E (the entrant) and I (the incumbent). At the initial node, E chooses between "Enter" and "Not." If "Not" is chosen, then the game ends, and the payoffs are 0 for E and 2 for I. If "Enter" is chosen, then I chooses between A (accommodate) and F (fight). If A is taken, the payoffs are 1 for both players. If F is taken, the payoffs are -1 for both players.

1. What is the set of subgame-perfect equilibria? What is the set of Nash equilibria?

2. Consider the infinitely-repeated version of this game with discount factor $\delta \in (0, 1)$, where the same incumbent plays every period, while different entrants play at different periods. Each entrant is indexed by the period she moves, t. That is, the set of players is $\{I, E_1, E_2, E_3, \dots\}$. Suppose that the entire extensive-form is that of perfect information. Note that if E_t plays "Not" at period t, then no player moving at period t' such that $t' > t$ would observe the intended action by the incumbent at period t.

   Prove or disprove each of the following claims.

   (a) There is a pure subgame-perfect equilibrium such that, on the path of play, every entrant enters and the incumbent always accommodates.

   (b) There exists $\delta' < 1$ and a pure subgame-perfect equilibrium such that, for all $\delta \in (\delta', 1)$, on the path of play, no entrant enters.

   (c) For any infinite sequence of actions in the extensive form such that I never fights, there exists $\delta' < 1$ and a pure subgame-perfect equilibrium such that, for all $\delta \in (\delta', 1)$, that sequence realizes on the path of play.