Theory field exam
9 August 2017

There are two questions on this exam, one for the directed reading course and another for Economics 206. Answer all parts for both questions.

Question 1 (reading course)

(a) Let X be a finite set of consequences and S be a finite set of states. Fix some set of priors $C \subset \Delta S$ and an affine (expected-utility) function $v : X \to \mathbb{R}$. Consider the binary relation $\succsim$ on $(\Delta X)^S$ defined by $f \succsim g$ if and only if there exists some $p \in C$ such that

$$\int_S v \circ f \, dp \geq \int_S v \circ f \, dp.$$

Observe that $\succsim$ is generally intransitive. Prove or provide a counterexample to the following statements:

(i) $\succsim$ has convex upper contour sets if and only if C is a singleton.

(ii) $\succsim$ satisfies independence, in the sense that $f \succsim g$ if and only if $\alpha f + (1-\alpha)h \succsim \alpha g + (1-\alpha)h$.

(b) Sketch the main steps in the proof of the DLR (2001) main theorem. You do **not** have to be very explicit. The main objective is to specify the domains on which the Riesz Representation Theorem and the Hahn-Banach Theorem are applied.

(Question 2 is on the next page.)

Question 2 (Econ 206)

The econ department could higher one of two candidates $x = a$ or $x = b$. Group A at the econ department *privately* observes the quality of candidate a given by θ_a and group B privately observes the quality θ_b of candidate b. θ_a, θ_b are independently, uniformly distributed on $[0, 1]$. The utility of group A/B if their candidate a/b is hired equals the quality of their candidate if the other candidate is hired it equals $\alpha \in [0, 1]$ times the utility of the other candidate, ie.

$$U_A \equiv \mathbf{1}_{\{x=a\}}\theta_a + \mathbf{1}_{\{x=b\}}\alpha\theta_b$$
$$U_B \equiv \mathbf{1}_{\{x=b\}}\theta_b + \mathbf{1}_{\{x=a\}}\alpha\theta_a \,.$$

We first assume that monetary transfers between the groups are impossible.

(a) Characterize the set of Pareto efficient allocations **without** monetary transfers.

(b) Characterize the set of allocations which maximize the utilitarian welfare (ie. the sum of agents utilites) **without** monetary transfers.

From now on we assume that transfers are possible and denote the transfer made by group A by t_A and the transfer made by group B by t_B. We assume that preferences are quasi-linear such that the overall utilities of the groups are given by

$$U_A - t_A$$
$$U_B - t_B \,.$$

We assume that all transfers made by the groups are collected by the department which has a utility of $t_A + t_B$.

(c) Characterize the set of Pareto efficient allocations **with** monetary transfers taking into account the departments utility.

(e) Define formally what a mechanism is in this context.

(f) Define formally what a direct mechanism is in this context.

(g) Define formally what an dominant strategy incentive compatible (direct) mechanism is in this context.

(h) Derive all mechanisms which implement the Pareto efficient allocation with transfers in dominant strategies. Prove that the mechanisms you found implement the Pareto efficient allocation and no other mechanism implements it.

(h) Show that there exists no budget balanced mechanism ($t_A + t_B = 0$) which implements the Pareto efficient allocation with transfers in dominant strategies. (Hint: The cases where both candidates have a quality of zero, both have a quality of one, and one has a quality of one and the other's quality zero will together lead to a contradiction.)