Theory field exam
9 August 2019

There are two questions on two pages for this exam, Question A for Economics 207 and Question B for Economics 206. Answer all parts for both questions.

Question A (Economics 207)

1. Let S be a finite state space.[1] Construct a set of priors $C \in \Delta S$ on a state space S such that preferences represented by the utility function $U : [0,1]^S \to \mathbb{R}$ defined by

$$U(f) = \min_{p \in C} \int_S f \, dp$$

do not satisfy comonotonic independence, as defined below. Sketch an argument for why that representation will fail comonotonic independence.

Definition 0.1. A binary relation $\succsim$ on $[0,1]^S$ satisfies **comonotonic independence** if, for all $\alpha \in (0,1)$,

$$f \succsim g \Longleftrightarrow \alpha f + (1-\alpha)h \succsim \alpha f + (1-\alpha)h$$

whenever f, g, h are pairwise comonotonic. Recall f and g are comotononic if $[f(s) - f(t)] \cdot [g(s) - g(t)] \geq 0$ for all s, t.

2. Maintain the notation from the part (1). Consider the following "α-maxmin" representation:

$$U(f) = \alpha \min_{p \in C} \int_S f \, dp + (1-\alpha) \max \min_{p \in C} \int_S f \, dp$$

where $\alpha \in [0,1]$. Prove or provide counterexamples to the following **true or false** claims. Substantial credit will be given for the correct answer ("true" or "false") without a complete proof or counterexample.

   (a) If $\succsim$ has an α-maxmin representation, then $\succsim$ satisfies C-independence, as defined below.

   Definition 0.2. A binary relation $\succsim$ on $[0,1]^S$ satisfies **C-independence** if, for all $\alpha \in (0,1)$,

   $$f \succsim g \Longleftrightarrow \alpha f + (1-\alpha)x \succsim \alpha f + (1-\alpha)x$$

   whenever $f, g \in [0,1]^S$ and x is a constant act in $[0,1]$.

   (b) If $\succsim$ has an α-maxmin representation, then $\succsim$ satisfies uncertainty aversion, as defined below.

   Definition 0.3. A binary relation $\succsim$ on $[0,1]^S$ satisfies **uncertainty aversion** if, for all $\alpha \in (0,1)$,

   $$f \sim g \Longrightarrow \alpha f + (1-\alpha)g \succsim f$$

   whenever $f, g \in [0,1]^S$.

3. Suppose $\succsim$ admits a self-control representation in GP (2001). Prove or provide a counterexample to the following statement: $\succsim$ satisfies Indifference to Randomization, that is, any closed set A is indifferent to its convex hull.

[1] To ease exposition and avoid the step of converting to utility-vectors, we will implicitly assume the agent is risk-neutral over wealth on the unit interval $[0,1]$. Alternatively, we can imagine there are only two consequences and $[0,1]$ parameterizes lotteries by the probability of the more desirable deterministic consequence.

Question B (Economics 206)

There are n agents indexed by $i \in \{1, \ldots, n\}$ and each agent has a type $\theta_i \in [0,1]$ which is independently distributed according to the CDF $F : [0,1] \to [0,1]$ with strictly positive density f. Agents are privately informed about their type. There are also n objects with commonly known qualities $(q_1, \ldots, q_n) \in [0,1]^n$. The designer can allocate at most one object to each agent. Agent i's value from getting allocated object k equals

$$\theta_i \, q_k \, .$$

We assume that agents have quasi-linear preferences and their utility when they pay t_i is given by

$$\begin{cases} \theta_i q_k - t_i & \text{if agent } i \text{ gets object } k \\ -t_i & \text{if agent } i \text{ does not receive an object} \end{cases} .$$

Participation is voluntary such that no agent can get a utility less than 0.

1. Characterize the set of dominant strategy incentive compatible direct mechanisms.
2. Characterize the set of Bayes Nash incentive compatible direct mechanisms.
3. Characterize the set of Pareto efficient allocations of objects to agents without transfers.
4. Characterize the set of Pareto efficient allocations of objects to agents with transfers.
5. Derive the utilitarian efficient allocation of objects to agents (i.e. the allocation that maximizes the sum of the agents' physical utilities ignoring transfers).
6. Derive a dominant strategy incentive compatible mechanism that implements the utilitarian efficient allocation of objects to agents.
7. Characterize all such mechanisms.
8. Derive the sellers revenue in a given Bayes Nash incentive compatible mechanism.
9. Derive the revenue maximizing mechanism assuming that the designer assigns a value of zero to every object.