Field Exam in Advanced Economic Theory

Q1.

Consider a two period bargaining model in which an uninformed buyer makes offers. If there is trade in period 1 the payoffs are $\alpha q - p$ for the buyer and p - q for the seller where p is the price and q is the quality and $\alpha > 0$. We assume the realization of q is only known to the seller and drawn from a Uniform (0,1). If instead they trade in the second period their payoffs from a time zero perspective are $\delta(\alpha q - p)$ and $\delta(p - q)$ for $\delta \in [0, 1]$ and if they don't trade they get 0.

- a) Show that the equilibrium will be characterized by two cutoff types $q_1(\alpha, \delta) \in [0, 1]$ and $q_2(\alpha, \delta) \in [q_1, 1]$ such that types $q < q_1$ trade in the first period and that types q such that $q_1 \leq q \leq q_2$ trade in second period and finally types above q_2 do not trade.
- b) How does welfare relative to the full information welfare depend on α and δ ? (If you find this hard to do analytically, at least work out a few examples with low and high values and explain)
- c) Assume $\alpha=1.7$ and $\delta=0.8$ and suppose that with probability γ in the second period another buyer arrives in the market in which case, they both simultaneously submit offers to the seller. How does welfare relative to the full information welfare depend on γ ? (Do only the $\gamma=1$ case if you are short on time.)

Please show your work carefully and explain clearly your findings. If possible try to give an economic intuition for your results.

Q2.

Consider a two-period model of consumption for a continuum of households indexed on the unit interval, $i \in [0, 1]$. In period one, household i consumes c_{1i} . In period two, household i consumes c_{2i} . The social planner has the following preferences over consumption in period 1 and 2:

$$\theta_i u(c_{1i}) + \delta v(c_{2i}),$$

where θ_i is a taste shifter for household i, δ is a discount factor, and u and v are strictly increasing and strictly concave. Further assume that u' and v' converge to ∞ as their respective arguments fall to zero. Finally, assume that θ has bounded support with closure $[\theta, \overline{\theta}]$, where $0 < \theta$.

a. This type of simple two-period model can be used to gain some insights about policies relating to retirement savings. In that context, briefly discuss how the parameter θ can be used to account for "shocks" that affect the optimal consumption path over the lifetime.

Now suppose that the individuals making consumption decisions during period 1 share the preferences of the social planner, but discount differently, such that the individuals' preferences at period 1 are given by:

$$\theta_i u(c_{1i}) + \beta \delta v(c_{2i}),$$

where $0 \le \beta \le 1$.

b. Discuss how this formulation relates to the work on time inconsistency in economics. Specifically: What are some of the seminal papers that have used this basic structure to model time inconsistency? Does the concept of naivite vs sophistication come into play in this two-period setup?

Assume that $\delta = 1$, and $u = v = \ln(x)$, such that the social planner's preferences are given by:

$$\theta_i ln(c_{1i}) + ln(c_{2i}),$$

while the individuals' preferences at period one are given by:

$$\theta_i ln(c_{1i}) + \beta ln(c_{2i}),$$

Futher, assume that each individual has lifetime resources of 1 that can be costlessly distributed across periods, such that for each individual i we have the constraint:

$$c_{1i} + c_{2i} = 1.$$

- c. Solve for the optimal consumption profile (c_1, c_2) given the social planner's preferences as a function of the realized taste shock θ .
- d. Solve for the actual consumption profile generated by the consumption decisions in period 1 of the individual as a function of θ and β .
 - e. Compare your answers in c and d and briefly discuss any differences.

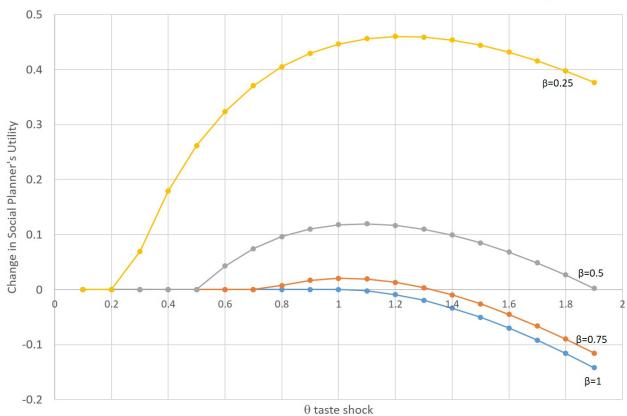
Suppose that the social planner has a technology to create a completely illiquid retirement account that can only be accessed for consumption in period 2. The social planner can allocate any fraction of the lifetime resources to that account. The individual in period 1 is then free to choose period 1 consumption up to but not exceeding the remaining resources. That is, in period 1 there is no way to borrow from the resources allocated to the retirement account, but it is possible to consume less than the remaining non-retirement resources in period 1. Further we

assume that the social planner cannot observe θ for an individual, and as such cannot make the retirement allocation (i.e., period 2 allocation) a function of the realized θ .

f. Consider a case in which $\beta = .5$ for all individuals and $\theta = .5$ or $\theta = 1$, each with probability of 0.5. Show that from the perspective of the social planner's preferences that it will increase expected utility if the social planner allocates 1/2 of resources to the illiquid retirement account that can only be accessed for period 2 consumption.

More generally whether the social planner wants to place limits on liquidity for the first-period decision can depend on the distributions of both β and θ . The graph below shows how the social planner's utility changes going from unconstrained choice by the period 1 self (i.e., your consumption-profile solution in part d) to a situation where 50% of resources are allocated for consumption in period 2 as described above. The graph shows the utility change the social planner has at the individual level for different levels of θ and shows 4 lines representing the change for different levels of β .

Change in Social Planner's Utility when 50% of Resources Allocated to Illiquid Retirement Account versus Unconstrained Choice by Period 1 Self for different combinations of $\boldsymbol{\theta}$ and $\boldsymbol{\beta}$.



- g. Discuss the lines for $\beta=1$ and $\beta=.75$ in the graph above. Specifically, explain why the line for $\beta=1$ is zero and then falls negative for $\theta>1$. Further discuss how the $\beta=.75$ line helps to highlight the tradeoffs inherent in limiting liquidity for the period 1 self.
- h. Now consider the lines for $\beta = .5$ and $\beta = .25$. What is the intuition for why the move to allocating 50% of resources to period 2 consumption (weakly) increases social-planner welfare regardless of the realization of θ (at

least over this range of θ)?

i. Finally, consider a social planner who has to make a decision about whether or not to enact the "50% share in an illiquid retirement account allocated for period 2" policy for a population with heterogeneous β . Based on the graph above, speculate thoughtfully about how the social planner's decision is likely to be affected by heterogeneity in β across the population.