Theory field exam
Winter 2012
Problem for 209A

There are two questions for this problem. Answer both.

1. A **bargaining problem** is a pair $\langle S, d\rangle$ where $S \subset \mathbb{R}^2$ is compact and convex, $d \in S$ and there exists $s \in S$ such that $s_i > d_i$ for $i = 1, 2$. The set of all bargaining problems $\langle S, d\rangle$ is denoted by $\mathbf{B}$. A **bargaining solution** is a function $f : \mathbf{B} \to \mathbb{R}^2$ such that f assigns to each bargaining problem $\langle S, d\rangle \in \mathbf{B}$ a unique element in S.

   For any $S \in \mathbf{B}$, define $\bar{s}_i \equiv \max_{s\in S} s_i$ for $i = 1, 2$. A bargaining solution f on $\mathbf{B}$ is said to be *strongly monotonic* if for any $S, T \in \mathbf{B}$ such that

   $$\frac{\bar{t}_2}{\bar{s}_2} = \frac{\bar{t}_1}{\bar{s}_1}$$

   and $f(S) \in T$ imply that $f(T) \geq f(S)$.

   Show that the bargaining solution $f : \Sigma \to \mathbb{R}^2_+$ given by

   $$f(S, d) = \left\{ \frac{s_1}{\bar{s}_1} = \frac{s_2}{\bar{s}_2} : s \in S \right\} \cap SPO(S)$$

   is the only strongly monotonic and strongly Pareto optimal (SPO) bargaining solution on $\mathbf{B}$. To simplify, assume that all all bargaining problems are comprehensive (a set $X \in \mathbb{R}^2_+$ is comprehensive if $x \in X$ and $0 \leq y \leq x$ then $y \in X$).

2. Suppose there are two (2) bidders whose values for an object are independently drawn from the uniform distribution on $[0, 1]$. They are engaged in the α**-average** price auction, i.e. the winning bidder i (that is, the bidder who submits the highest bid x_i) pays the $\alpha x_i + (1-\alpha)x_{-i}$ (that is, α of her own bid and $1 - \alpha$ of her opponent's bid) where $\alpha \in [0, 1]$.

   (a) Find an equilibrium bidding function $\beta_i(x_i)$ for this auction, and prove that it is an equilibrium.

   (b) Compute the seller's expected revenue in this auction.

   (c) Prove or provide a counterexample to the following statement: The variance (not the expectation!) of the revenue is decreasing in α. You may consider specific values (for example, $\alpha = 0, 1$) if the general case is too difficult.

Problem from 219A:

This problem considers the role of reference dependence in distributional social preferences. It has eight parts (and three different characters).

Casi has the following social preferences. (Since we will add in gain-loss utility, these aren't her complete preferences, but just her "consumption-utility" component.)

In allocating between herself and another party, Casi has preferences:

$$M = \pi_{other} + \pi_{self} \text{ if } \pi_{self} \geq \pi_{other}$$
$$M = 2 \cdot \pi_{self} \text{ if } \pi_{self} \leq \pi_{other}$$

Another way to write Casi's consumption-utility function is:

$$M = Min[\pi_{other} + \pi_{self}, 2 \cdot \pi_{self}].$$

Note that, kink notwithstanding, Casi's consumption utility function is continuous and "unitary": in terms of how this fits into a reference-dependent model, she has only a single consumption dimension.

Casi also has two-part-linear gain-loss utility with respect to this consumption-utility, with expectations as her reference point. She has utility only in the period where she implements her choice, with no prospective gain-loss utility, anticipatory utility, etc. The parameters for her gain-loss component of her utility are $\eta = 1$ and $\lambda = 3$.

a) Suppose Casi had expected the allocation $(\pi_{other}, \pi_{self}) = (10, 10)$ for sure, but instead at the last minute was given the choice between $(\pi_{other}, \pi_{self}) = (10, 10)$ and a 50/50 lottery, delivering $(\pi_{other}, \pi_{self}) = (0, X)$ with probability .5, and delivering $(\pi_{other}, \pi_{self}) = (X, 0)$ with probability .5. For which values of X will Casi take the lottery?

b) Suppose instead that Casi had expected the 50/50 lottery, delivering $(\pi_{other}, \pi_{self}) = (0, X)$ with probability .5, and delivering $(\pi_{other}, \pi_{self}) = (X, 0)$ with probability .5, and then at the last minute was given the choice between keeping the lottery or taking the the allocation $(\pi_{other}, \pi_{self}) = (10, 10)$ for sure. For which values (if any) of X will Casi keep the lottery?

c) Suppose that Casi had long expected the choice between $(\pi_{other}, \pi_{self}) = (10, 10)$ and a 50/50 lottery, delivering $(\pi_{other}, \pi_{self}) = (0, X)$ with probability .5, and delivering $(\pi_{other}, \pi_{self}) = (X, 0)$ with probability .5, and cannot commit to her choice ahead of time. For which values (if any) of X will Casi take the lottery?

d) Suppose that Casi had long expected the choice between $(\pi_{other}, \pi_{self}) = (10, 10)$ and a 50/50 lottery, delivering $(\pi_{other}, \pi_{self}) = (0, X)$ with probability .5, and delivering $(\pi_{other}, \pi_{self}) = (X, 0)$ with probability .5, and *can* commit to her choice ahead of time. For which values (if any) of X will Casi take the lottery?

Now consider Alexander. Alexander has two dimensions to his consumption utility function:

$$m_1 = 2 \cdot \pi_{self}$$
$$m_2 = -Max[\pi_{self} - \pi_{other}, 0]$$

Note that this means that Alexander has overall consumption utility function of

$$M = Min[\pi_{other} + \pi_{self}, 2 \cdot \pi_{self}].$$

Like Casi, Alexander also has two-part-linear gain-loss utility with respect to this consumption utility, with expectations as his reference point. His parameters are likewise $\eta = 1$ and $\lambda = 3$. But Alexander assesses gains and losses separately on each of his two dimensions.

e) Suppose Alexander had expected the allocation $(\pi_{other}, \pi_{self}) = (10, 10)$ for sure, but instead at the last minute was given the choice between $(\pi_{other}, \pi_{self}) = (10, 10)$ and a 50/50 lottery, delivering $(\pi_{other}, \pi_{self}) = (0, X)$ with probability .5, and delivering $(\pi_{other}, \pi_{self}) = (X, 0)$ with probability .5. For which values (if any) of X will Alexander take the lottery?

f) Suppose instead that Alexander had expected the 50/50 lottery, delivering $(\pi_{other}, \pi_{self}) = (0, X)$ with probability .5, and delivering $(\pi_{other}, \pi_{self}) = (X, 0)$ with probability .5, and then at the last minute was given the choice between keeping the lottery or taking the the allocation $(\pi_{other}, \pi_{self}) = (10, 10)$ for sure. For which values (if any) of X will Alexander keep the lottery?

Now (and finally) consider Bertil. Bertil has two dimensions to his consumption utility function, just like Alexander. But Bertil's dimensions are different. They are:

$$m_1 = \pi_{other} + \pi_{self}$$
$$m_2 = -Max[\pi_{other} - \pi_{self}, 0]$$

Note that this means that Bertil has overall consumption utility function of

$$M = Min[\pi_{other} + \pi_{self}, 2 \cdot \pi_{self}].$$

Like Casi and Alexander, Bertil also has two-part-linear gain-loss utility with respect to this consumption-utility, with expectations as his reference point. His parameters are likewise $\eta = 1$ and $\lambda = 3$. Like Alexander but unlike Casi, Bertil assesses gains and losses separately on each of his two dimensions.

g) Suppose Bertil had expected the allocation $(\pi_{other}, \pi_{self}) = (10, 10)$ for sure, but instead at the last minute was given the choice between $(\pi_{other}, \pi_{self}) = (10, 10)$ and a 50/50 lottery, delivering $(\pi_{other}, \pi_{self}) = (0, X)$ with probability .5, and delivering $(\pi_{other}, \pi_{self}) = (X, 0)$ with probability .5. For which values (if any) of X will Bertil take the lottery?

h) Suppose instead that Bertil had expected the 50/50 lottery, delivering $(\pi_{other}, \pi_{self}) = (0, X)$ with probability .5, and delivering $(\pi_{other}, \pi_{self}) = (X, 0)$ with probability .5, and then at the last minute was given the choice between keeping the lottery or taking the the allocation $(\pi_{other}, \pi_{self}) = (10, 10)$ for sure. For which values (if any) of X will Bertil keep the lottery?