

Labor Economics

There are three parts of the exam. Please answer all three parts. You should plan to spend about one hour per question. *Use equations and graphs whenever possible, but be sure to explain your notation.* You may use a calculator for arithmetic if you wish.

PLEASE WRITE YOUR ANSWERS FOR EACH PART IN A SEPARATE BOOK.

PART I

1. Carefully derive the elasticity version of the Slutsky equation for labor supply, relating the uncompensated elasticity of labor supply to the compensated elasticity and the marginal propensity to consume out of unearned income. Hint: use the expenditure function.

2. Consider an individual who will work from the current period (t) to some future period T , faces a sequence of real wages $\{w_t, w_{t+1}, \dots, w_T\}$ and has current (real) assets A_t . Assume the individual has a per-period utility function $u(c_t, h_t; a_t)$ where $\{a_t\}$ is a sequence of preference shifters (known by the individual). Assume in addition that the individual discounts the future at a constant rate β , and can earn a real interest rate r_t on savings from period t to $t+1$. Finally, assume that $h_t > 0$ in all periods, so we can ignore the extensive margin, and that the individual has no sources of income other than labor earnings.

a) Write down the Bellman equation defining the value of an optimal lifecycle plan starting in period t , $V_t(A_t)$.

b) Use the first order conditions for the optimal choices in period t to show that the **Frisch demand functions** can be written in terms of w_t , a_t , r_t and $\lambda_t = V'_t(A_t)$.

c) Using your answer in (b), show that the change in the optimal choice of hours from period $t-1$ to t can be written as:

$$\Delta \log h_t = \alpha + \eta \Delta \log w_t + \varepsilon_t \quad (*)$$

where η is the elasticity of Frisch labor supply with respect to wages. Carefully discuss the "error term" ε_t . In particular, carefully enumerate the terms included in this "error term" and their likely relationship with $\Delta \log w_t$.

d) Discuss three different approaches that have been used in the literature to estimate

equation (*): (a) using instruments for $\Delta \log w_t$; (b) using the change in consumption as a control function; (c) using information on anticipated future changes in income.

2. Suppose that economy-wide real output (y_t) depends on inputs of capital (K_t) and various types of labor $L_{1t}, L_{2t}, \dots, L_{Jt}$:

$$y_t = f(K_t, L_{1t}, L_{2t}, \dots, L_{Jt}).$$

a) Assume that

$$f(K, L_1, L_2, \dots, L_K) = AK^{1-\alpha} L^\alpha,$$

$$\text{where } L = h(L_1, L_2, \dots, L_K)$$

and that the price of capital, r , is constant. Show in this case that output is linear in the labor aggregate L , and does not depend on the inputs (or wages) of any particular subgroup of labor.

b) Katz and Murphy assume that there are two types of labor in every period (t), high education labor L_{1t} and low education labor L_{2t} , and that $h(\cdot)$ is CES:

$$h(L_{1t}, L_{2t}) = [\theta_{1t} L_{1t}^\rho + \theta_{2t} L_{2t}^\rho]^{1/\rho}$$

where $\rho = (\sigma - 1)/\sigma$ and σ is the elasticity of substitution between education groups. Suppose that there is **skill-biased technical change**, of the form

$$\log \theta_{1t}/\theta_{2t} = \phi + \lambda t$$

where $\lambda > 0$. Show in this case that the log relative wages of more educated workers will include a trend component and a term that depends on relative employment of high versus low educated workers (L_{1t}/L_{2t}).

c) Suppose that there are 2 types of educated workers, males ("M") and females ("F"), and also 2 types of low educated workers. Assume that h is a "nested CES"

$$h(L_{1t}, L_{2t}) = [\theta_{1t} L_{1t}^\rho + \theta_{2t} L_{2t}^\rho]^{1/\rho}$$

$$L_{1t} = [a_M L_{1Mt}^\tau + a_F L_{1Ft}^\tau]^{1/\tau},$$

$$L_{2t} = [b_M L_{2Mt}^\tau + b_F L_{2Ft}^\tau]^{1/\tau}$$

Show in this case that the relative wages of highly educated males to low educated

males depends on the overall ratio of high to low education workers of both genders (L_{1t}/L_{2t}) and on the gender-specific ratio L_{1Mt}/L_{2Mt} .

d) Suppose one is interested in whether males and females in the same education group are "perfect substitutes". What does "perfect substitutes" imply about the parameter τ in part (c). How could you test the hypothesis of "perfect substitutes"?

PART II

You are analyzing data from a randomized trial of a new vaccine. In this experiment, half of a sample of 1,000 patients was randomly offered access to the vaccine at a subsidized price. Some of the patients offered access to the vaccine decided not to use it and some who were not offered the subsidized price still decided to get vaccinated. Twenty years later mortality records were used to assess whether the patients were still alive. Summary data from this experiment are given in the following tabulation along with baseline information on demographics.

Offered	Vaccinated	Obs	% Survived	% Female
N	N	400	.8	.4
N	Y	100	.7	.6
Y	N	100	.9	.4
Y	Y	400	.8	.5

a) Propose a test statistic for evaluating the null hypothesis that being offered the vaccine is independent of baseline gender. Can you reject this null hypothesis at conventional levels of significance? State any auxiliary assumptions underlying your analysis.

b) Compute an estimate of the LATE of vaccine usage on survival.

c) Compute estimates of the fractions of the population that are “compliers”, “always takers”, and “never takers”.

d) Compute estimates of the fraction female among “compliers”, “always takers”, and “never takers”. Are women more or less likely to respond to the vaccine offer than men?

e) Compute estimates of the mean survival rate when vaccinated of compliers, the mean survival rate when not vaccinated of compliers, the mean survival rate when vaccinated of “always takers”, and the mean survival rate when not vaccinated of “never takers”.

f) Suppose now that the experiment generated less compliance, resulting in sample proportions given by the revised table:

Offered	Vaccinated	Obs	% Survived	% Female
N	N	260	.8	.4
N	Y	240	.7	.6
Y	N	240	.9	.4
Y	Y	260	.8	.5

Why does the evidence in this table suggest that the exclusion restriction underlying the LATE model is likely to be violated? Suggest an approach to formally testing the exclusion restriction.

PART III

Consider a competitive industry in which a large number of profit-maximizing firms hire workers and rent capital to produce output. Production requires one worker and one unit of capital, and renting a unit of capital costs $c > 0$. Workers are heterogeneous: worker talent, θ , is uniformly distributed on $[0, 1]$. A firm that hires a worker of talent θ produces output equal to θ . Talent is initially unknown to both workers and firms, and output is publicly observed. A worker can work in this market for at most 2 periods, and cannot commit in advance to work for the same firm for more than one period.

There is free entry of firms and an unlimited supply of potential workers, all of whom have opportunity costs of 0 outside the market. Workers and firms are risk neutral. Inverse demand for the industry's output is $P^d(Q)$, where P is price and Q is quantity.

1. Consider an equilibrium in which both novice workers (who have never worked before) and some veteran workers (who have previously worked for one period) are hired. Let w_1 denote the wage paid to novice workers, and $w_2(\theta)$ denote the wage paid to a veteran worker of talent θ

A. Explain why w_2 is a function of θ but w_1 is not.

B. Show that this equilibrium must involve a "cutoff" value of talent, θ^* , such that veterans with talent above θ^* will work in the market for a second period, and veterans with talent below θ^* will not.

C. What wage will a veteran with the cutoff talent level θ^* be paid in equilibrium? Why?

D. Use the fact that firms must make zero profits on veterans with talent θ^* to derive an expression for the equilibrium output price, P , as a function of the production cost c and the cutoff talent θ^* .

E. Find an expression for the wage paid to employed veteran workers, $w_2(\theta)$, as a function of c and θ^* .

F. Derive the novice wage, w_1 , as a function of θ^* and c . Show that this wage must be negative. Interpret this finding.

G. Derive the equilibrium value of θ^* .

2. Now suppose that novice workers are prohibited from borrowing, so they cannot accept negative wages.

A. Derive the new equilibrium cutoff θ^* .

B. Compared to the equilibrium in part (1), is average talent in the industry now higher or lower? For veterans of a given talent level, are wages higher or lower? Give some intuition.

C. Discuss a contract or policy remedy that might restore the unconstrained equilibrium in part (1). Is this remedy likely to be feasible in real-world labor markets? Why or why not?