

Labor Economics

August 2018

There are four questions on this exam. **Please answer three out of four.** You should plan to spend about one hour per question. **Please write your answers for each question in a separate book.**

Question 1: The Value of Human Capital

Debates over human capital policies often center on the “rate of return” to early life investments. Suppose children grow up to have utility over lifetime consumption c and leisure \bar{l} given by the function $u(c, \bar{l})$. The lifetime budget constraint of a child with human capital level w can be written:

$$c = w(T - \bar{l}) + b,$$

where T is a time endowment and b is unearned income.

- Use the above notation to define an “excess expenditure” function $e(\omega, \bar{u})$ giving the minimal level of unearned income necessary to obtain utility level \bar{u} at wage level ω .
- Write the compensating variation associated with raising the child’s human capital level from w_0 to w_1 in terms of the excess expenditure function. Provide a verbal description of what this compensating variation metric measures.
- Derive an approximation to the compensating variation, appropriate for the case where $w_1 - w_0$ is small, in terms of the (baseline) *Marshallian* labor supply $l(w_0, b)$.
- Derive the impact of a small increase in human capital on total earnings in terms of $l(w_0, b)$ and the uncompensated labor supply elasticity $\varepsilon \equiv \frac{w_0}{l(w_0, b)} \frac{\partial l(w_0, b)}{\partial w}$.
- Use your answers above to evaluate the following statement:

The dollar value to an individual of a small increase in human capital is approximately equal to the dollar value of the additional earnings such an intervention yields.

- Given your knowledge of the empirical labor supply literature, about how much should a young man be willing to pay to retain a human capital investment that will raise his lifetime earnings by \$10,000? Is your answer the same for a young woman?

Question 2: Binary Choice

Consider a binary choice model of the form

$$D_i = 1 \{X_i' \beta > U_i\}, \quad U_i | X_i \sim \text{Uniform}(0, 1).$$

The vector $X_i = (1, X_{i1}, \dots, X_{iK})$ includes a constant and K and other variables, some of which may be continuous with unbounded support.

- Derive the probability that $D_i = 1$ conditional on X_i , given by $Pr[D_i = 1 | X_i]$.
- Derive the average marginal effect of a change in X_{ik} on the conditional probability that $D_i = 1$, given by $E[\partial Pr[D_i = 1 | X_i] / \partial X_{ik}]$.
- You have access to a sample of N *iid* observations on (D_i, X_i) . Propose a maximum likelihood estimator (MLE) of the parameter β . Propose an estimator of the average partial effect from part (b).

d) As an alternative to MLE, your colleague suggests estimating the marginal effects of X_i via an ordinary least squares (OLS) regression of D_i on X_i . Discuss conditions under which the OLS coefficient vector is consistent for β . If these conditions are satisfied, how does the asymptotic efficiency of OLS compare to the efficiency of the MLE from part (c)? If these conditions are not satisfied, how would you interpret the OLS coefficients?

e) Suppose your conditions from (d) are satisfied. Propose a two-step weighted least squares estimator of β that is asymptotically equivalent to the MLE.

Question 3: Oligopsony

Consider a labor market with only two employers. Suppose workers $i \in \{1, \dots, N\}$ have indirect utility over firms $j \in \{1, 2\}$ given by:

$$U_{ij} = \beta \ln w_j + \ln A_j + \varepsilon_{ij}$$

where (w_1, w_2) are firm specific wage offers, (A_1, A_2) are firm specific non-wage amenities, and $\{\varepsilon_{i1}, \varepsilon_{i2}\}_{i=1}^N$ are worker specific tastes over non-wage amenities. Assume that N is very large and that the $\{\varepsilon_{i1}, \varepsilon_{i2}\}_{i=1}^N$ are iid draws from a Type I extreme value distribution.

a) Derive an expression for the fraction of workers that will work at firm 1 as a function of the two wages and amenity levels.

b) What is the elasticity of firm 1's employment share with respect to the wage ratio w_1/w_2 ? How does this elasticity depend upon firm 1's market share?

c) Suppose that each additional worker yields p dollars of extra revenue for firm 1. The firm cannot observe $\{\varepsilon_{i1}, \varepsilon_{i2}\}_{i=1}^N$ but knows the joint distribution of these errors. Assume also that firm 1 knows (A_1, A_2) and that $\beta = 1$. Finally, suppose firm 1 believes its wage offer will not influence the wage of firm 2 (i.e., w_2 is exogenous). What wage will firm 1 offer? (Hint: you will need to use the quadratic formula)

d) Now suppose firm 1 conjectures that the elasticity of firm 2's wage with respect to its own wage is $1/2$ (i.e., $\frac{d \ln w_2}{d \ln w_1} = 0.5$). What does firm 1 now believe is the elasticity of its employment share with respect to w_1 ? How would you expect this conjecture to affect its wage offer?

Question 4: Simultaneous Equations

Consider the following simultaneous equations model relating an outcome Y_i to a scalar endogenous variable X_i and a scalar instrument Z_i , with no intercept:

$$Y_i = \beta X_i + \epsilon_i,$$

$$X_i = \pi Z_i + \eta_i.$$

Assume $E[Z_i] = E[\epsilon_i] = E[\eta_i] = 0$ and $Cov(\epsilon_i, Z_i) = Cov(\eta_i, Z_i) = 0$.

a) Derive a "reduced form" equation relating Y_i to the instrument Z_i . How does the reduced form coefficient on Z_i depend on the underlying structural parameters? How does the covariance between the reduced form and first stage errors depend on these parameters?

b) Suppose a sample of N iid observations on (Y_i, X_i, Z_i) is available. Define ordinary least squares (OLS) estimators of the model's reduced form and first stage parameters. Write down an expression for the joint asymptotic distribution of these two OLS estimates, treating the Z_i 's as fixed.

c) Suppose you are interested in testing the null hypothesis $H_0 : \beta = 0$. Based on your results from (a) and (b), propose a test of this hypothesis that uses only the estimated reduced form.

d) Propose an estimator of the structural coefficient β constructed from the reduced form and OLS estimates. Use your answer to part (b) and apply the delta method to derive the asymptotic distribution of this estimator.

- e) Use your result in part (d) to propose an alternative test of the null hypothesis $H_0 : \beta = 0$.
- f) Suppose the first stage coefficient π is “small.” Which of your two tests of $H_0 : \beta = 0$ do you expect would have better finite-sample properties? Why?