University of California, Berkeley — Department of Economics
Field Exam — January 2019

Labor Economics

There are three parts of the exam. Please answer all three parts. You should plan to spend about one hour per question. *Use equations and graphs whenever possible, but be sure to explain your notation.* You may use a calculator for arithmetic if you wish.

PLEASE WRITE YOUR ANSWERS FOR EACH PART IN A SEPARATE BOOK.

Part I.

1. Consider an individual who faces an sequence of real wages in the current and future periods, $w_t, w_{t+1}, .. w_T$, and has current (real) assets A_t. (For simplicity assume that the only source of income is labor income). Assume the individual has a per-period utility function $u(c_t, h_t; a_t)$ where $\{a_t\}$ is a sequence of preference shifters over the lifecycle. Assume in addition that the individual discounts the future at a constant rate $\beta<1$, and can earn a real interest rate r_t on savings from period t to $t+1$.

a) Write down the Bellman equation defining the value of an optimal lifecycle plan starting in period t, $V_t(A_t)$.

b) Define $\lambda_t \equiv V'_t(A_t)$. What is the relationship between λ_t and $E_t[\lambda_{t+1}]$?

c) Explain what is meant by "the intertemporal elasticity of labor supply" in the context of this model.

c) Suppose that

$$u(c_t, h_t; a_t) = \varphi(c_t) - \frac{a_t \eta}{1+\eta} h^{\frac{1+\eta}{\eta}} ,$$

i.e. within-period utility is separable in consumption and leisure, and preference shocks do not affect the marginal utility of consumption. Show that the intertemporal elasticity of labor supply in this case is constant. What is its value?

d) Using the assumptions of part (c), show that the change in the optimal choice of hours from period to $t-1$ to t can be written as:

$$\Delta \log h_t = \alpha + \eta \Delta \log w_t + \varepsilon_t .$$

What terms are included in ε_t? Discuss the likely biases in an OLS approach for estimating η.

e) Suppose that professors have non-stochastic and constant real wages. Assuming that preferences are as described in part (c), what would you have to assume to about the preference shocks to explain the phenomenon of "retirement", where people stop working and don't return to work later?

2. Suppose that economy-wide real output (y_t) depends on inputs of capital (K_t) and various types of labor L_{1t}, L_{2t}, ... L_{Jt}:

$$y_t = AK^{1-\alpha} L^{\alpha} \quad , \text{ where } L = h(L_1, L_2, \ldots L_K) \text{ .}$$

a) Show that if the price of capital in period t, $r_{t,}$ is constant then output is linear in the "labor aggregate" L, and does not depend on the inputs of any particular subgroup of labor.

b) Suppose that there are only two types of labor, skilled and unskilled, and that h( ) is CES:

$$h(L_{1t}, L_{2t}) = [\ \theta_{1t} L_{1t}^{\rho} + \theta_{2t} L_{2t}^{\rho}\ ]^{1/\rho} \text{ .}$$

Explain what is meant by "**skill-biased technical change**" in the context of this model.

c) Assuming the model of part (b), explain how one could estimate ρ using data on relative wages and relative employment of groups 1 and 2. Carefully explain what you are assuming about θ_{1t} and θ_{2t} .

d) Suppose that there are 2 types of skilled workers, male ("M") and female ("F"), and also two types of unskilled workers, male and female. Assume that h is a "nested CES"

$$h(L_{1t}, L_{2t}) = [\ \theta_{1t} L_{1t}^{\rho} + \theta_{2t} L_{2t}^{\rho}\ ]^{1/\rho}$$
$$L_{1t} = [\ a_{Ft} L_{1Ft}^{\tau} + a_{Mt} L_{1Mt}^{\tau}\ ]^{1/\tau} \quad ,$$
$$L_{2t} = [\ b_{Ft} L_{2Ft}^{\tau} + b_{Mt} L_{2Mt}^{\tau}\ ]^{1/\tau}$$

Show how you can estimate the parameters ρ and τ using data on wages and employment of the various skill groups. What do you have to assume about the terms a_{Ft}, a_{Mt}, b_{Ft}, and b_{Mt} in your proposed strategy?

d) An economist has suggested that over the past 3 decades, technical changes have made women more productive than men. What would you expect to see in the data if this were true?

PART II

Consider a labor market in which workers have heterogeneous ability. Worker ability takes values $\theta \in [\underline{\theta}, 1]$, and is distributed continuously on this interval. In a first period, workers observe their ability and choose an education level $e \in [0, \bar{e}]$. The cost of education for a worker of ability θ is $c(e, \theta) = \lambda e/\theta$ with $\lambda > 0$. In a second period, a large number of firms compete a la Bertrand to hire workers. Firms observe education but not ability before making wage offers. Productivity for a worker with ability θ and education e is $y(e, \theta) = \theta$.

1. Give some intuition for the functional forms of $c(e, \theta)$ and $y(e, \theta)$. What do these functions imply about the role of schooling in the labor market? What possibilities do these functions rule out?
2. Define a Perfect Bayesian Equilibrium in this model.
3. Write down the problem that determines $e^*(\theta)$, the optimal choice of e for a worker of ability θ. Use this problem to derive a condition relating education, wages and ability in a separating equilibrium.
4. What must be true of firm wage offers in equilibrium? Use your answer to derive a second condition relating education, wages and ability.
5. Use your answers in parts (3) and (4) to derive an expression for $\partial e^*(\theta)/\partial\theta$, the slope of the equilibrium education/ability relationship.
6. Assuming that $e^*(\underline{\theta}) = 0$, solve for $e^*(\theta)$.
7. Use your results from part (6) to write an expression for $w(e)$, the equilibrium wage for a worker with education e.
8. Briefly discuss empirical evidence in favor or against the view of schooling underlying the model in this question.

PART III

A large literature reviewed in Solon (1992) considers estimation of intergenerational earnings elasticities (IGEs). Recently, a debate has emerged regarding "which" elasticity to estimate. Suppose we have a dataset $\{X_i, Y_i\}_{i=1}^N$ giving the lifetime earnings of N father-son pairs, with X_i being the father's lifetime earnings and Y_i the son's lifetime earnings. While the traditional approach has been to estimate an OLS regression of the form:

$$\ln Y_i = \alpha + \beta \ln X_i + \varepsilon_i,$$

Mitnik et al. (2014) have instead suggested fitting a pseudo-maximum likelihood Poisson regression model that imposes the conditional mean restriction:

$$\ln E\left[Y_i|X_i = x\right] = \alpha + \beta x$$

To think about the difference between these approaches, suppose that:

$$Y_i|X_i \overset{iid}{\sim} F_{Y|X}\left(.\right).$$

Assume the earnings distributions of fathers and sons have no mass points and that everyone works at some point in their lifetime (i.e., earnings are strictly positive). We can define the τ'th conditional quantile of earnings among sons whose fathers earn x as:

$$q\left(x, \tau\right) \equiv F_{Y|X=x}^{-1}\left(\tau\right)$$

1) Prove that we can therefore write:

$$Y_i = q\left(X_i, U_i\right),$$

where $U_i|X_i \sim \text{Uniform}(0,1)$.

2) Assuming that the conditional quantile function $q\left(x, \tau\right)$ is differentiable in both its arguments, we can define the quantile-specific intergenerational elasticity function:

$$\sigma\left(x, \tau\right) \equiv \frac{dq\left(x, \tau\right)}{dx}\frac{x}{q\left(x, \tau\right)},$$

which summarizes how each quantile of son's earnings depend upon his father's earnings when the father's earnings are x. Evaluate the following three derivatives in terms of $\sigma\left(x, \tau\right)$:

a) $\frac{d}{d\log x}E\left[Y_i|X_i = x\right]$

b) $\frac{d}{d\log x}\log\left(E\left[Y_i|X_i = x\right]\right)$

c) $\frac{d}{d\log x}E\left[\log Y_i|X_i = x\right]$

3) Describe a situation where you would expect $\frac{d}{d\log x}\log E\left[Y_i|X_i = x\right] > \frac{d}{d\log x}E\left[\log Y_i|X_i = x\right]$.

4) Describe a possible drawback of using $\int \frac{d}{d\log x}\log E\left[Y_i|X_i = x\right] dF_X\left(x\right)$ as the preferred IGE concept.

5) Describe a possible drawback of using $\int \frac{d}{d\log x}E\left[\log Y_i|X_i = x\right] dF_X\left(x\right)$ as the preferred IGE concept.

6) Chetty et al. (2014) examine a "rank-rank" IGE specification of the form:

$$E\left[F_Y\left(Y_i\right)|F_X\left(X_i\right) = p\right] = \alpha + \beta p$$

where $F_Y\left(.\right)$ is the CDF of son's earnings and $F_X\left(.\right)$ the CDF of father's earnings. Derive an expression for $\frac{d}{dp}E\left[F_Y\left(Y_i\right)|F_X\left(X_i\right) = p\right]$ in terms of the derivative $\Delta\left(p, \tau\right) \equiv \frac{dq\left(F_X^{-1}(p), \tau\right)}{dF_X^{-1}(p)}$. Interpret your answer.