UNIVERSITY OF CALIFORNIA
Department of Economics **August 2014**

MACROECONOMICS FIELD EXAM

ANSWER ALL QUESTIONS

Part One is intended to take about 50 minutes, and Part Two is intended to take about 1 hour and 50 minutes. You have 3 hours.

PART ONE

1. This question asks you to use a Solow-style model to investigate some ideas that have been discussed in the context of Thomas Piketty's recent work.

Consider a continuous-time economy. Output at time t is given by $Y(t) = F\big(K(t), A(t)L(t)\big)$. The notation is standard. $F(\cdot)$ satisfies the usual assumptions, including constant returns to scale. Thus, we can write $y(t) = f\big(k(t)\big)$, where $y \equiv Y/(AL)$, $k \equiv K/(AL)$, and $f(k) \equiv F(k, 1)$. $f(\cdot)$ satisfies the Inada conditions.

$K(0)$, $A(0)$, and $L(0)$ are given and are all strictly positive. The dynamics of the inputs are given by $\dot{K}(t) = [Y(t) - C(t)] - \delta K(t)$; $\dot{A}(t) = gA(t)$; and $\dot{L} = nL(t)$, where $\delta > 0$, $g > 0$, and $n > 0$.

Where the model differs from the Solow model is in its assumption about the determinants of C. Factors are paid their marginal products, and all labor income is consumed and all other income is saved. Thus, $C(t) = L(t)\left[\frac{\partial Y(t)}{\partial L(t)}\right]$.

Assume that the initial conditions are such that $\partial Y/\partial K$ at $t = 0$ is strictly greater than $n + g + \delta$.

a. Show that the properties of the production function imply that the capital-output ratio, K/Y, is rising if and only if the growth rate of K is greater than $n + g$ – that is, if and only if k is rising.

b. Describe the qualitative behavior of the capital-output ratio over time. (For example, does it grow or fall without bound? Gradually approach some constant level from above or below? Something else?) Explain your reasoning.

c. Many popular summaries of Piketty's work describe his thesis as: Since the return to capital exceeds the growth rate of the economy, the capital-output ratio tends to grow without bound. By assumption, this economy starts in a situation where the return to capital exceeds the economy's growth rate. If you found in (b) that K/Y grows without bound, explain intuitively whether the driving force of this unbounded growth is that the return to capital exceeds the economy's growth rate. Alternatively, if you found in (b) that K/Y does not grow without bound, explain intuitively what is wrong with the statement that the return to capital exceeding the economy's growth rate tends to cause K/Y to grow without bound.

d. Suppose $F(\cdot)$ is Cobb-Douglas. Describe the qualitative behavior over time of the share of *net* capital income (that is, $K(t)\left[\frac{\partial Y(t)}{\partial K(t)} - \delta\right]$) in *net* output (that is, $Y(t) - \delta K(t)$). Explain your reasoning. Is the common statement that an excess of the return to capital over the economy's growth rate causes capital's share to rise over time correct in this case?

PART ONE (continued)

Question 2

Let $y_t(s^t)$ be the aggregate endowment for this economy, where $s^t = (s_0, s_1, ..., s_t)$ with s_t is a real-valued random variable. The economy is populated by a continuum of identical agents with preferences over consumption given by

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t u(c_t(s^t)) Pr_t(s^t)$$

where Pr_t is the probability over histories s^t, for any t and $u : \mathbb{R}_+ \to \mathbb{R}$ is increasing, twice continuously differentiable and strictly concave and satisfies the inada conditions.

The agents can save/borrow at rate $R \geq 1$. At any time t and history s^t the budget constraint is given by

$$c_t(s^t) + Ra_{t+1}(s^t) \leq a_t(s^{t-1}) + y_t(s^t)$$

also $a_{t+1}(s^t) \geq -\phi$.

1. Derive the first order conditions of this problem.

Suppose that $y_t(s^t) = y_t \geq 0$ for all t. That is, y_t is deterministic.

2. (i) How does the optimal sequence of consumption behave if $\phi = \infty$? (ii) How does the optimal sequence of consumption behave if $\phi = 0$?

3. Suppose $R\beta = 1$ and $\phi = 0$. (i) Suppose $y_t = 1/(1+t)$, how does the optimal sequence of consumption look like? (ii) Suppose $y_t = 1$ if $t \leq T$ and $y_t = 2$ otherwise, how does the optimal sequence of consumption look like?

4. Suppose $R\beta = 1$ and $\phi = 0$. Does the optimal sequence of consumption diverge as time goes to infinity? Explain why.

Suppose now that $y_t(s^t) = s_t$ and s_t is i.i.d drawn from P.

5. Suppose $R\beta = 1$ and $\phi = 0$. Does the optimal sequence of consumption diverge (almost surely) as time goes to infinity? Explain why.

PART TWO.

Question 1

Consider an infinitely lived government that is confronted with a stochastic endowment $(y_t)_{t=0}^{\infty}$ with y_t i.i.d. drawn from P with $support(P) = \{y^1, y^2, ..., y^k\}$. The government's utility function over consumption is given by $E[\sum_{t=0}^{\infty} \beta^t u(c_t(y^t))|y_0]$ where $u : \mathbb{R} \to \mathbb{R}$ is continuous, bounded and increasing.

At any time t, the government can issue one-period non-state contingent bonds, but is unable to commit to pay them. It will honor its debt only if the value of repayment exceeds the value of default.

If at time t, the government decided to service the outstanding debt given by B_t, its budget constraint is given by

$$c_t + B_t \leq q(y_t, B_{t+1})B_{t+1} + y_t$$

where $(y, B) \mapsto q(y, B) \in [0, 1] \subseteq \mathbb{R}$ is the price profile of government debt and is assumed to be decreasing in B and $\frac{\partial q(y,B)B}{\partial B}\Big|_{B=0} > 0$ and $\lim_{B\to\infty} q(y, B)B = 0$.

If at any given time, the government decided not to service the outstanding debt, then it will enter a period of financial autarky. While in this period, the government is forced to run a balance budget: $c = ay$ (i.e., cannot issue bonds) where $a \in (0, 1)$ represents output costs of being in autarky. At any time, the government will exit financial autarky with probability $\lambda \in (0, 1)$. When the government exits financial autarky, it regains access to financial markets and starts with zero outstanding debt.

The problem of the government consists of choosing sequences of consumption, debt and default decisions to maximize its utility, satisfying the budget constraint.

1. What is the state of the economy?

2. (i) Write down the Bellman equation for this problem. (ii) What additional assumptions (if any) you need to argue that the Principle of Optimality (PO) holds. **Hint:** You can ignore measurability issues when verifying the assumptions.

3. (i) Show that the value function exists in the space of uniformly bounded functions. (ii) Is the value function continuous in B? (iii) Is the value function monotonic on B? **Hint:** Feel free to use whatever lemma/theorem/etc you need without proving them (but make sure to verify that their assumptions are met).

4. Go as far as you can characterizing the optimal default decision. In particular, is it true that if the government chooses to default for (g, B) it will do so too for (g, B') with $B' \geq B$? If yes, prove it.

PART TWO (continued)

2. Illiquid mortgage debt vs. liquid cash holdings

Consider the following model related to Kaplan and Violente (2014) framework:

Household i has *log* preferences over utility flows from ***non-durable consumption*** C_{it} and discounts future at rate ρ:

$$U_i = E\left[\int_0^{\infty} e^{-\rho t}\frac{C_{it}^{1-\gamma}}{1-\gamma}dt\right]$$

Labor supply is fixed. ***Labor income*** process is *exogenous* and follows a Geometric Brownian Motion (rules out *negative* income realizations).

$$dY_{it} = \mu_Y Y_{it}dt + \sigma_Y Y_{it}dZ_{it}^Y,\ t > 0$$
$$Y_{i0} = y_i > 0$$

Household i has two type of assets: ***Mortgage debt*** B_{it} with interest rate r_B and ***cash holdings*** $W_{it} \geq 0$ which earns a rate of return $r_f < r_B$. Let's call X_{it} household $\boldsymbol{i's}$ (flow of) ***mortgage payment***. Then the law of motion for not refinanced mortgage debt is:

$$\begin{aligned} dB_{it} &= \left[rB_{it} - X_{it}\right]dt \\ X_{it} &\geq rB_{it} \end{aligned}$$

Which means that the household can pay down her mortgage as quickly as she likes, but the debt balance cannot grow over time, unless the household refinances the mortgage. Therefore, without refinancing household cash holdings (W_{it}) evolves according to:

$$dW_{it} = \left[Y_{it} - C_{it} + r_f W_{it} - X_{it}\right]dt$$

Finally, the household has the option to refinance its mortgage at any amount $\tilde{B}_i(\leq \theta QH_i)$ after it pays a one time fixed cost Φ(where θ is the maximum loan to value ratio and QH_i is the value of the house the household lives in).

1. Describe intuitively why the household may hold some cash and have a mortgage debt at the same time?
2. Let's assume the household does not refinance the mortgage during the period $[\tau, \tau + s]$ and the cash holdings are positive during this period. What is the law of motion for consumption during this period?
3. Derive the law of motion for W_{it} when the household exercises the refinancing option. (Note that there will be a discontinuity in household's cash holding just before and after the refinancing.)

   Now let's call $\mathbf{V}(B_{it}, W_{it}, Y_{it})$ the value function for a household with mortgage debt B_{it}, cash holdings W_{it} and income Y_{it}.

4. When is it optimal for the household to make a mortgage payment in excess of the required minimum payment? (i.e. $X_{it} > rB_{it}$) (Hint: think about the partial derivatives of V with respect to B and W)
5. When is it optimal for the household to refinance the mortgage? (Hint: use your answer to part 2 and think about changes in B and W just before and just after refinancing)
6. What happens for consumption of the household just after the household refinance the mortgage?
7. What is the impact of the reduction in the fixed cost of refinancing on average cash holdings?

PART TWO (continued)

3. The short-run of increases in government purchases. (In answering this question, please be sure to devote at least as much attention to part (b) as to part (a).)

a. Explain briefly the predictions concerning the short-run macroeconomic effects of a temporary, unanticipated increase in government purchases of: (a) a baseline real-business-cycle model; (b) a baseline old Keynesian model; (c) a baseline new Keynesian model. Do the predictions depend importantly on what is assumed about the conduct of monetary policy? Explain your answers. (It is not necessary to write down or solve a model in each category. Clear explanations of the models' predictions and the mechanisms underlying the predictions are sufficient.)

b. In your view, which type of model is the empirical evidence about the short-run effects of changes in government purchases most consistent with? Defend your answer. Be as specific as you can in discussing papers and their strengths and weaknesses.