Labor Field Exam

January 2022

Instructions: There are three parts to this exam. Each part will be weighted equally. Calculators are not necessary. Please explain your notation in order to maximize chances of partial credit.

Advice: If you cannot answer a question move on to the next question. Each part should take less than one hour to solve. You do not need to answer the parts in the order that they are listed below.

Part I - ECON 244

A judge must decide whether to grant a defendant bail ($d = 1$) or not ($d = 0$). Our judge is concerned with whether the defendant will flee if granted bail, which we represent by the indicator $Y = 1\{\text{flees bail}\}$. The judge has loss function

$$L(d, Y) = d \cdot Y \cdot \kappa + (1 - d) \cdot c$$

where c gives the cost of pre-trial detention and $\kappa > c$ is the cost of fleeing bail. To ease the decisionmaking process, the county machine learning department has issued a risk score S obeying $\Pr(Y = 1|S = s) = s$ for all $s \in [0, 1]$.

a) Suppose the only information the judge observes about the defendant is their score S. Derive the judge's Bayes risk function $R(d) = \mathbb{E}[L(d, Y)|S]$.

b) Based on your answer to part a), what is the judge's Bayes decision rule?

c) The machine learning department reports that they discovered a small error in the risk score algorithm. They assure the judge however that $\max_{s\in[0,1]} |\Pr(Y = 1|S = s) - s| \leq \varepsilon$. Derive the maximal risk function $\bar{R}(d)$ faced by the judge.

d) What is the minimax decision rule?

e) Suppose that the risk score S was constructed based upon the flight risk of previous defendants who were granted bail. That is, $S(x) = \Pr_T(Y = 1|D = 1, X = x)$ where $\Pr_T$ refers to the probability in a historical training period, D refers to the bail decisions of other judges in that training period, and X is a vector of discrete defendant characteristics. Suppose the judges in the training period had access to the same scores $S = S(X)$ as in the present period and all followed the Bayes decision rule given in your answer to part b). Assume further that the joint distribution of defendant flight risks and characteristics has not changed since the training period – i.e., that $\Pr(Y = 1, X) = \Pr_T(Y = 1, X)$. Are scores available for all defendants? Is the judge able to make optimal decisions?

f) Now suppose the judge also sees a binary signal Z before making their decision. The judge believes

$$\Pr(Z = 1|Y = 1, S = s) = s + \delta$$

$$\Pr(Z=1|Y=0,S=s)=s$$

Use Bayes rule to derive the posteriors $\Pr(Y=1|S=s,Z=1)$ and $\Pr(Y=1|S=s,Z=0)$.

g) Write the Bayesian decision rule for granting bail as a function of $\Pr(Y=1|S=s,Z=z)$. What can we infer about the Z of a defendant who would be granted bail under this rule but not the decision rule in your answer to b)?

h) (Bonus) By the law of total probability

$$\begin{aligned}\Pr_T(Y=1|S=s,D=1) &= \Pr_T(Y=1|S=s,D=1,Z=1)\Pr_T(Z=1|S=s,D=1)\\ &+ \Pr_T(Y=1|S=s,D=1,Z=0)\left[1-\Pr_T(Z=1|S=s,D=1)\right].\end{aligned}$$

Suppose judges in the training period made decisions based upon S and Z with the beliefs described in part f). Because their decision only depended upon (S,Z), it follows that $Pr_T(Y=1|S=s,D=1,Z=z)=Pr_T(Y=1|S=s,Z=z)$. What can we say about how $Pr_T(Z=1|S=s,D=1)$ compares to $Pr_T(Z=1|S=s)$? Are risk scores positively or negative selected?

Part II - ECON 250A

An infinitely-lived agent chooses duration of schooling S to maximize his/her present discounted value of earnings. The agent is born at time zero and earns nothing until s/he leaves schooling and starts working. If the agent is working, earnings at time t are given by $y_t(S)$. There is no direct cost of schooling. The agent can freely borrow and save at interest rate r.

A. Write down an expression for the present discounted value of the agent's earnings at time zero, treating time as continuous.

B. Suppose potential earnings at every time are given by $y_t(S)=\alpha+\beta S$. Find S^*, the agent's optimal duration of schooling. Interpret your result.

C. Suppose a population of individuals make choices according to your solution in part B. These individuals have the same values of α and β but face different interest rates r. You run a regression of observed earnings on schooling in this population. Interpret the slope coefficient on schooling obtained from this regression. How does this parameter relate to the parameters of individuals' potential earnings functions? Give some intuition.

D. Now suppose potential earnings at time t are given by $y_t(S)=\exp(\gamma t)\times(\alpha+\beta S)$, where $0<\gamma<r$. Explain why earnings might take this form. Solve for the optimal schooling choice S^*.

E. Suppose a population of individuals make choices as in part D. The parameters α and β are the same for all individuals, but γ and r may differ across individuals. You run a regression of earnings at a particular time on schooling in this population. How would you expect the resulting slope coefficient to relate to the parameters of the potential earnings functions?

F. Relate your answer from part (E) to empirical strategies for estimating the returns to schooling, referencing relevant empirical literature.

Part III - ECON 250B

Consider a setting where there are two risk-neutral workers and one risk-neutral manager. The workers choose their own levels of effort a_i and produce output x_i that is a noisy function of their effort:

$$\begin{aligned}x_1 &= a_1+\epsilon_1\\ x_2 &= a_2+\epsilon_2\end{aligned}$$

Where the errors ϵ_1 and ϵ_2 are independent, and identically distributed with mean 0 and variance σ^2. The manager is only able to observe workers' output (i.e. not their effort).

The workers have a convex cost of effort function $c(a)$ and have outside option $\bar{H}$.

1. [1 point] What is the efficient (first-best) level of effort? What is the expected total level of output produced when all workers exert the first-best level of effort?

2. [1 point] **Piece-rates:** Suppose the manager decides to pay each worker using the same piece-rate function. What wage (if any) implements the first-best?

3. **Ranking Workers** Suppose instead the manager decides to pay workers based on the ranking of their output. The worker who produces more receives $\bar{w}$ and the worker who produces less receives $\underline{w}$.

   (a) [1 point] What are some advantages and disadvantages of this scheme?

   (b) [1 point] Suppose worker 2 exerts effort a_2. What is worker 1's expected payoff?

   (c) [1 point] Use $F(\cdot)$ to denote the CDF of $(\epsilon_2 - \epsilon_1)$, with corresponding density $f(\cdot)$. Write the first-order condition characterizing the level of effort that maximizes the worker's utility. Interpret this condition.

   (d) [1 point] Note that, in a symmetric equilibrium, both workers exert the same levels of effort $a_1^* = a_2^*$. In order for both workers to exert the first best level of effort, what must $\bar{w}$ and $\underline{w}$ be? Interpret these conditions. Hint: remember workers' participation constraints!

   (e) [1 point] Consider a setting where workers have a different cost of effort function $c_2(a)$ where $c_2'(a) < c'(a)\ \forall a$. How does the "spread" $(\bar{w} - \underline{w})$ in this setting compare to that in the base case?

   (f) [1 point] Suppose a new firm enters the market, raising workers' outside options to $\bar{H}^{new} > \bar{H}$. How does this affect the optimal choice of $\bar{w}$ and $\underline{w}$? Assume that the manager still wants to employ both workers.

4. **Empirical Literature:** [2 points] Relate your answers to the above to at least one of the empirical papers discussed in class.